Inter Digital Transducer Modelling through Mason Equivalent Circuit Model
Design and Simulation

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Abstract—The frequency reliance of inter-digital transducer is analyzed with the help of MASON's Equivalent circuit which is based on Smith’s Equivalent circuit which is further based on Foster’s Network. An inter-digital transducer has been demonstrated as a RLC network. The circuit is simulated by Simulation program with Integrated Circuit Emphasis (HSPICE), a well-liked electronic path simulator. The acoustic wave devices are not suitable to simulation through circuit simulator. In this paper, an electrical model of Mason’s Equivalent electrical circuit for an inter-digital transducer (IDT) is projected which is well-suited with a broadly-cast-off universal resolution circuit simulator SPICE built-in out with the proficiency to simulate the negative capacitances and inductances. The investigation is done to prove the straightforwardness of establishing the frequency and time domain physical characteristics of the IDT and flexibility to simulate the IDT laterally with other peripheral circuit elements. Sensors with high Q for resonance provide a better stability.

Keywords—Mason, piezoelectric transducer, SAW, MEMS, Foster method, resonator

I. INTRODUCTION

There are numerous simulation design techniques for SAW devices like impulse response, coupling of modes, superposition theorem, field theory, etc. From these entire mentioned methods the equivalent circuit model provides superior results than other methods. Surface Acoustic Wave (SAW) devices are not compatible to simulation through circuit simulators because they are mainly electro-acoustic in nature, whereas any electrical circuit simulation requires pure electrical model. The paper intelligence significant improvements in the previous model and proposes it for HSPICE circuit simulation. On the other hand some finite element model (FEM) tools such as COMSOL provide 3D view simulation for SAW devices [1][2]. An inter-digital transducer is a comb shaped like metallic structure having a fixed gap between its fingers having the combination of both acoustic and electric field. The Mason’s equivalent electrical circuit is or the three port system shown in fig 1.

Fig.1. SAW IDT as a 3 port system
II. BACKGROUND

Figure 2 shows the equivalent circuit model for N number of fingers and figure 3 shows the Mason’s ECM for one pair of finger. However higher the count of fingers, the more time it take for simulation, and more computational resources are required. Then there is a problem to find a compromise allowing them to reduce the computational complexity of the desired FEM simulation. The second problem can occur when the fingers are too large. The similarities between the electrical and mechanical elements are described by W. P. Mason in [4]. As SAW devices are generally composed of several periodic sections, each section consists of a pair of fingers so Mason’s ECM can be easily applied on any one of the periodic sections [5].

\[
TANEQ = jZ_0 \tan \frac{\theta}{2}, \quad CSCEQ = jZ_0 \csc \theta, \quad \theta = \pi \frac{f}{f_0}
\]

\[
Z_0 = \frac{1}{k^2 C_S f_0}, \quad f_0 = \frac{V_0}{L}
\]

Here K is the electromechanical coupling coefficient or factor, C_s is electrode capacitance per finger pair, V_0 is the acoustic velocity for free region, L is the length of one pitch or period.
We know that a transmission line can be equated and designed as a distributed LC circuit and lumped circuit model as shown in Fig. 5(a) and (b). Reflect on a transmission line of length $l$. We wish to treat it as a two-port network and describe it in terms of its terminal parameters. One way is to use the amplitudes $V^+$ and $V^-$ of the waves travelling in the positive and negative $z$ directions, respectively. This gives us 4 quantities, $V^+_1, V^-_1, V^+_2$ and $V^-_2$; any two of these can be treated as dependent variables and expressed in terms of the other two through a 2*2 matrix. Hence selecting the outgoing wave amplitudes $V^-_1$ and $V^+_2$ as the dependent variables, the scattering matrix $[S]$ is given as:

$$[S] = \begin{bmatrix} 0 & e^{-jkl} \\ e^{jkl} & 0 \end{bmatrix}$$

A linear two-port network is completely described by a 2*2 matrix. It is frequently more convenient to use the voltage $V$ and the current $I$ rather than the wave amplitudes $V^+$ and $V^-$. 

$$V = V^+ + V^- \quad (3)$$

$$I = \frac{1}{Z_0} (V^+ - V^-) \quad (4)$$

Using the voltages as the dependent variables, we get the impedance matrix $[Z]$:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Now there is a point to visualise the equivalence between the lumped circuit and the transmission line segment by showing that their $[Z]$ matrices are equal[21].

From $[S]$ matrix we have

$$V^-_1 = e^{-jkl} V^-_2 \quad (5)$$

$$V^+_2 = e^{-jkl} V^+_1 \quad (6)$$

Hence

$$V_2 = V^+_2 + V^-_2 = e^{-jkl} V^+_1 + e^{jkl} V^-_1 = V_1 \cos kl - jZ_0 I_1 \sin kl \quad (7)$$

Similarly,

$$Z_0 I_2 = V^+_2 - V^-_2 = Z_0 I_1 \cos kl - jV_1 \sin kl \quad (9)$$

This gives us the transmission matrix:

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} \cos kl & -j Z_0 \sin kl \\ -j \sin kl & \cos kl \end{bmatrix} \begin{bmatrix} V_1 \\ I_1 \end{bmatrix}$$

Now the $[Z]$ matrix with some arrangements is given as

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} -j \cot kl & j \csc kl \\ j \csc kl & -j \cot kl \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Here we have chosen the reference direction for both $I_1$ and $I_2$ in the positive $Z$ direction.
III. MODELLING OF MASON EQUIVALENT CIRCUIT

The absolute geometry or the model is shown in fig 4(a) and (b). The SAW IDT by means of a phase length \( p \) of the same circuit corresponds substantially to the midpoint to midpoint spacing sandwich between two contiguous fingers each of width \( W \). \( \psi \) is the transit angle of one periodic length section and is given by \( \psi = \frac{\pi W}{w_0} \) where \( w_0 \) is the center frequency. The characteristic impedance \( R_0 \) of the transmission line is dependent on \( C_s \), the static capacitance per unit period.

The paper used Foster’s circuit which states that the location and behaviour of the zeros and poles, and the understanding of the impedance at one other frequency is appropriate to decide the complete network. Specifically, the elements \( j R_0 \tan(\psi / 2) \) and \( -j R_0 \cos ec \psi \) can be behaviorally given by networks, shown in fig 6 and 7. Totally the modules of the IDT equivalent circuit of Figure 4(a) are therefore known in terms of \( R, L \) and \( C \) and are required for circuit analysis in a simulator such as HSPICE. It is known that HSPICE considers negative inductance and capacitance values for circuit simulation to be valid [2,3].

IV. DESIGNING AND SIMULATION

Considering the 20 finger pair IDT made up of aluminum with a 1.5 \( \mu \)m thickness on 64\(^{\circ}\) Y-Z LiNbO\(_3\) with finger width or aspect ratio of 0.5 with an aperture of 0.020100257091454, operating at a center frequency of 850MHz. All of the circuit simulations are performed using HSPICE and MATLAB. Here \( C_{01} \) i.e. oxide capacitance is considered as 4.6 pF and the circuit admittance is 3.6152 mS. An IDT is having a total static capacitance of 0.09 pF. The full description is shown in fig. 8 and 9. T-network elements represent lossless transmission lines consisting of \( N \) periodic section which is cascaded acoustically and in parallel electrically. In the equivalent circuit model, the impedances \( TANEQ, CSCEQ, TAN0, TANM, CSC0, \) and \( CSCM \) are independent of frequency. The proposed model substitute the above terms by LC network obtained using Foster’s reactance theorem.
V. RESULTS AND DISCUSSION

A 20 finger pair IDT presentation has been simulated by HSPICE at a centre frequency of 850MHz with a 3 dB bandwidth of 38 MHz(fig. 10 and 11). The source impedance is taken out to be $50 \, \Omega$. For simulation, up to 80 finger pairs have been magnificently handled by HSPICE without any convergence problem. The frequency response curve is showing very high quality factor when it resonates on its centre frequency i.e. 850 MHz.
An HSPICE and MATLAB compatible Mason’s Macro model has been proposed. HSPICE is used as it can handle negative capacitances and inductances. The proposed model is showing highest amplitude at eigen frequency with a very high quality factor. The present model which is simulated by HSPICE and MATLAB is capable of managing the second order edge effects. From the simulation result of frequency response, very high quality factor is obtained. Sensors with high Quality factor Q for resonance provide a better stability.

REFERENCES


