Curvelets and their Future Applications

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ABSTRACT
Multiresolution methods are deeply related to image processing, biological and computer vision, scientific computing, etc. The curvelet transform is a multiscale directional transform, which allows an almost optimal nonadaptive sparse representation of objects with edges. Despite the fact that wavelets have had a wide impact in image processing, they fail to efficiently represent objects with highly anisotropic elements such as lines or curvilinear structures (e.g. edges). The reason is that wavelets are non-geometrical and do not exploit the regularity of the edge curve. The Curvelet transforms were developed as an answer to the weakness of the wavelet transform. Curvelets take the form of basis elements which exhibit high directional sensitivity and are highly anisotropic. These very recent geometric image representations are built upon ideas of multiscale analysis and geometry. Curvelets have also proven useful in diverse fields beyond the traditional image processing application. Another reason for the success of curvelets is the availability of fast transform algorithms which are available in non-commercial software packages following the philosophy of reproducible research.

KEYWORDS
Curvelets, Wavelets, Multiresolution methods, Image Processing, Computer Vision

1.INTRODUCTION
Most natural images/signals exhibit line-like edges, i.e., discontinuities across curves (so-called line or curve singularities). Although applications of wavelets have become increasingly popular in scientific and engineering fields, traditional wavelets perform well only at representing point singularities, since they ignore the geometric properties of structures and do not exploit the regularity of edges. Therefore, wavelet-based compression, denoising, or structure extraction become computationally inefficient for geometric features with line and surface singularities. For example, when downloaded compressed images or videos, often have a mosaic phenomenon (i.e., block artifacts along edges of the images). This mosaic phenomenon mainly results from the poor ability of wavelets to handle line singularities.

One of the primary tasks in computer vision is to extract features from an image or a sequence of images. The features can be points, lines, edges, and textures. A given feature is characterized by position, direction, scale and other property parameters. The most common technique, used in early vision for extraction of such features, is linear filtering, which is also reflected in models used in biological visual systems, i.e., human visual motion sensing. Objects at different scales can arise from distinct physical processes. This leads to the use of scale space filtering and multiresolution wavelet transform in this field. An important motivation for computer vision is to obtain directional representations which capture anisotropic lines and edges while providing sparse decompositions.

2.CURVELETS
Curvelets can be seen as an extension of wavelets for multidimensional data. They recently appeared, keeping the multi-resolution and localization aspects of the wavelets (Candès and Donoho, 2002; Do, 2001). They were initially designed for (non-seismic) image compression and denoising, whenever the data contains some geometrical structures. The key difference between wavelets and curvelets is that only curvelets are really directional: the basis functions have elongated shapes, the width being proportional to the square of the length at the fine scale. In order to overcome the missing directional selectivity of conventional two-dimensional discrete wavelet transforms, a multiresolution geometric analysis (MGA), named curvelet transform, was proposed. In the two-dimensional (2D) case, the curvelet transform allows an almost optimal sparse representation of objects with singularities along smooth curves.

Comparing the curvelet system with the conventional Fourier and wavelet analysis, the short-time Fourier transform uses a shape fixed rectangle in frequency domain, and conventional wavelets use shape-changing (dilated)but area fixed windows. By contrast, the curvelet transform uses angled polar wedges or angled trapezoid windows in frequency domain in order to resolve also directional features.

In 1999, an anisotropic geometric wavelet transform, named ridgelet transform, was proposed by Candes and Donoho. The ridgelet transform is optimal at representing straight-line singularities. Unfortunately, global straight-line singularities are rarely observed in real applications. In order to analyze


local line or curve singularities, a natural idea is to consider a partition of the image, and then to apply the ridgelet transform to the obtained sub-images. This block ridgelet based transform, which is named curvelet transform, was first proposed by Candès and Donoho in 2000. Apart from the blocking effects, however, the application of this so-called first-generation curvelet transform is limited because the geometry of ridgelets is itself unclear, as they are not true ridge functions in digital images.

A considerably simpler second-generation curvelet transform based on a frequency partition technique was proposed. Recently, a variant of the second-generation curvelet transform was proposed to handle image boundaries by mirror extension. Previous versions of the transform treated image boundaries by periodization. Here, the main modifications are to tile the discrete cosine domain instead of the discrete Fourier domain, and to adequately reorganize the data. The obtained algorithm has the same computational complexity as the standard curvelet transform. The second-generation curvelet transform has been shown to be a very efficient tool for many different applications in image processing, seismic data exploration, fluid mechanics, and solving PDEs (partial differential equations). From the mathematical point of view, the strength of the curvelet approach is their ability to formulate strong theorems in approximation and operator theory. The discrete curvelet transform is very efficient in representing curve-like edges. But the current curvelet systems still have two main drawbacks:

1. They are not optimal for sparse approximation of curve features beyond C^2 singularities.
2. The discrete curvelet transform is highly redundant.

The currently available implementations of the discrete curvelet transform (see www.curvelet.org) aim to reduce the redundancy smartly. However, independently from the good theoretical results on N-term approximation by curvelets, the discrete curvelet transform is not appropriate for image compression. The question of how to construct an orthogonal curvelet-like transform is still open here.

It should be noted that several other directional multi-resolution bases, such as wedgelets, bandlets, contourlets, shearlets, platelets, have been proposed independently to identify and restore geometric features. These geometric/directional wavelets are uniformly called X-lets. In particular, the directional-filter-bank based contourlet transform can be seen as a certain discrete form of the curvelet transform. For contourlets, there exists an orthogonal version that is faster than current discrete curvelet algorithms. But contourlet functions have less clear directional features than curvelets which leads to artifacts in denoising and compression.

3.PROPERTIES OF CURVELETS

We present here the basic properties of the curvelet functions which have found wide range of application. Curvelets have compact support in frequency domain. This support is a polar wedge or a trapezoid. The wedges are rotated around zero, such that the supports of all curvelets represent a tiling of the two-dimensional frequency domain.

Curvelet functions φ_{j,k,l} are usually indicated by three indices; j denotes the scale index, l the orientation index, and k ∈ Z^2 the location in time domain. The scale j denotes the distance 2^j of the support wedge from zero and the length 2^{2j} as well as the width 2^{j/2} of the polar wedge. The orientation l determines the rotation angle 2^{j/2} π l / 2 ∈ [0, 2π) of the wedge, where l = 0, ..., 4 · 2^{j/2} − 1. Finally, the location (k_1/2^j, k_2/2^{j/2}) with k = (k_1, k_2) ∈ Z^2 indicates the translation of the curvelet function in time domain.

The main properties of curvelets can be summarized as follows:

1. The family of curvelet functions forms a tight frame of L^2 (R^2). That means, each function f ∈ L^2 (R^2) has a representation μ = ⟨f, φ_{j,k,l}⟩. The coefficients C_{j,k,l} := μ = ⟨f, φ_{j,k,l}⟩ are called curvelet coefficients.

2. The transform that computes the sequence of curvelet coefficients from f ∈ L^2 (R^2), is called curvelet transform. A fast curvelet transform can be realized by computing first the Fourier transform of f (by an FFT-algorithm), and computing the scalar product ⟨f, φ_{j,k,l}⟩ in frequency domain, using the small compact support of φ_{j,k,l}, see [2].

3. Curvelets are well-localized in time- and frequency domain. Because of their shape, they possess a high directional sensitivity.

4. Curvelets are constructed by tiling of the frequency plane, they are complex functions. One can construct also real curvelet functions by adding two curvelets that are supported in frequency domain on two polar wedges being symmetric with respect to zero.

5. Curvelets possess an infinite number of directional moments. This property implies that, if the essential support of the curvelet φ_{j,k,l} lies in a smooth part of f, then the corresponding curvelet coefficient C_{j,k,l} will be small, while, if the essential support of φ_{j,k,l} is aligned with an edge of f, then C_{j,k,l} will be significant.

These properties of the curvelet frame are essential for its ability to detect wave front sets efficiently.

4.CURVELETS IN IMAGE PROCESSING

Image processing is any form of signal processing for which the input is an image, such as a photograph or video frame; the output of image processing may be either an image or, a set of characteristics or parameters related to the image. Most image-processing techniques involve treating the image as a two-dimensional signal and applying standard signal-processing techniques to it.

Published papers in journals and conferences have shown that researchers have successfully applied curvelets for image denoising, image contrast enhancement, fusion of satellite images, motion estimation and video tracking, and surface characterization. Other work has studied the adaptability of the curvelet transform for different tasks of computer vision, such
as image retrieval, texture analysis, and object recognition. The curvelet transform implementations offer exact reconstruction, stability against perturbations, ease of implementation and low computational complexity. With the existing theory being put into practical use the application of Curvelets in image processing has offered exceptional improvement over wavelets in visual performance.

The image shown below shows a comparison of image being denoised using a curvelet transform.

CONCLUSION

This paper offers a comprehensive insight into the array of applications where Curvelets are now being applied to. With more vehement research and expansion of scope, Curvelets can be applied to diverse fields so that revolutionary results can be obtained.

FUTURE SCOPE

1) The computational cost of curvelets is higher than that of wavelets. Software that would be able to implement fast curvelet algorithm that reduce the computational time have continued to be a burgeoning area of research. The application of Curvelets in 3D has also become a promising area of research.

2) Currently, the curvelets are constructed in Fourier domain. There is no explicit space-domain formulation for curvelets. This brings troubles in many applications such as numerical modeling of PDEs. How to build a space-domain formulation of curvelets remains a challenge.

REFERENCES


